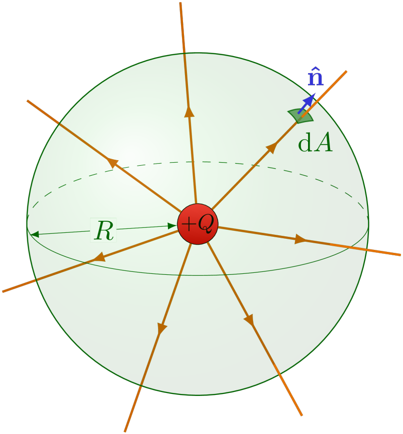
Coulomb's law:

$$\vec{F}_{qQ} = q\left(\frac{Q}{4\pi\epsilon_0}\cdot\frac{\vec{e}_{qQ}}{r_{qQ}^2}\right)$$

Vector property of the point $\vec{r}_q$ yielding the force on a charge q generated by a charge Q.

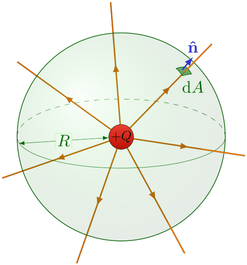
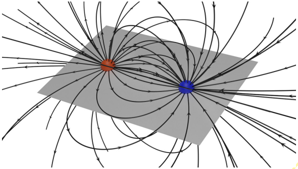
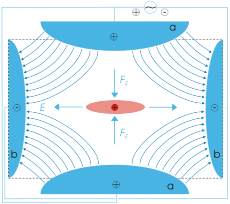
$\updownarrow$

electric field of Q



Static charge configurations:

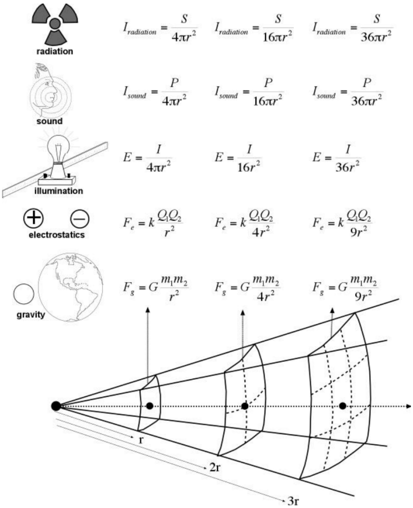
$$\vec{E}(q) = \frac{Q}{4\pi\epsilon_0}\cdot\frac{\vec{e}_{qQ}}{r^2_{qQ}} \xrightarrow{1\to N} \sum_{i=1}^{N}\frac{Q_i}{4\pi\epsilon_0}\cdot\frac{\vec{e}_{qQ_i}}{r^2_{qQ_i}} \xrightarrow{N\to\infty} \int_{\text{space}}\frac{\rho\,(\vec{q'})}{4\pi\epsilon_0}\cdot\frac{\vec{e}_{qq'}}{r^2_{qq'}}\,d^3r_{q'}$$

point charge	set of charges	charge density
		

Horribly complicated!
Universal characteristics of $\vec{E}_\rho$?

Flux conservation – inverse-square sources

What does this feature imply for the flux density of the E-field?

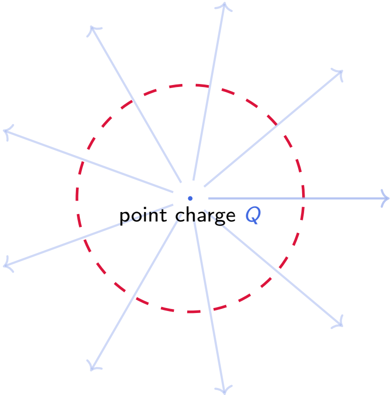


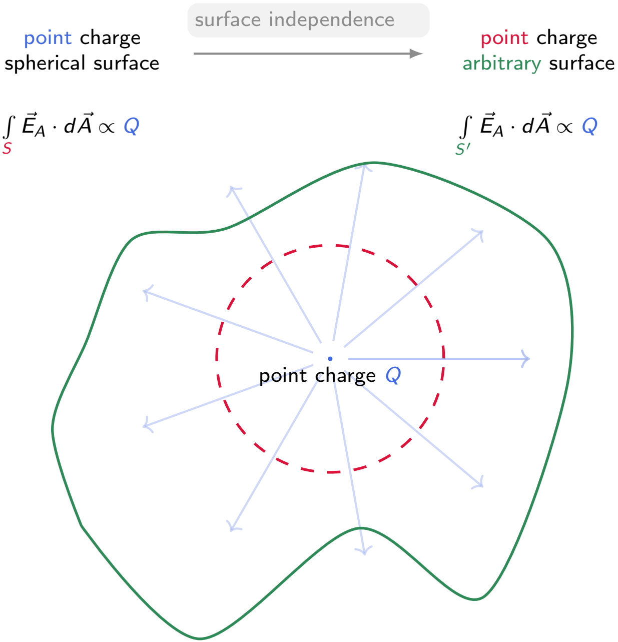
Whatever "substance" the source emits, its intensity decays at a rate inverse to the area of an enclosing spherical surface.

ECCE! Under which circumstances does radioactive radiation, sound, light obey the $1/r^2$ decay law?

point charge
spherical surface

$$\int_S \vec{E}_A \cdot d\vec{A} \propto Q$$





point charge
spherical surface
surface independence
point charge
arbitrary surface
$\int_S \vec{E}_A \cdot d\vec{A} \propto Q$
$\int_{S'} \vec{E}_A \cdot d\vec{A} \propto Q$
point charge Q

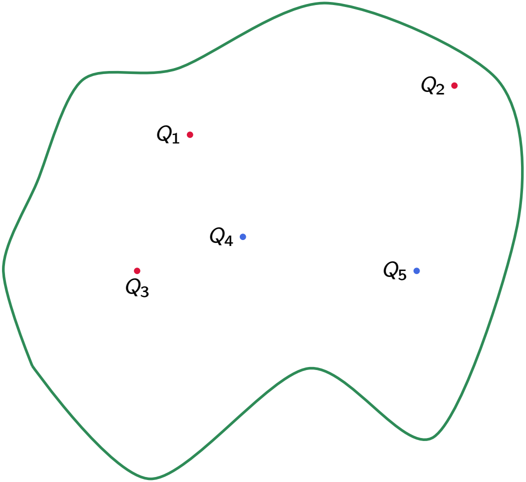
superposition (discrete)

point charge
arbitrary surface

$$\int_{S'} \vec{E}_A \cdot d\vec{A} \propto Q$$

point charge set
arbitrary surface

$$\int_{S'} \vec{E}_A \cdot d\vec{A} \propto \sum_i Q_i$$



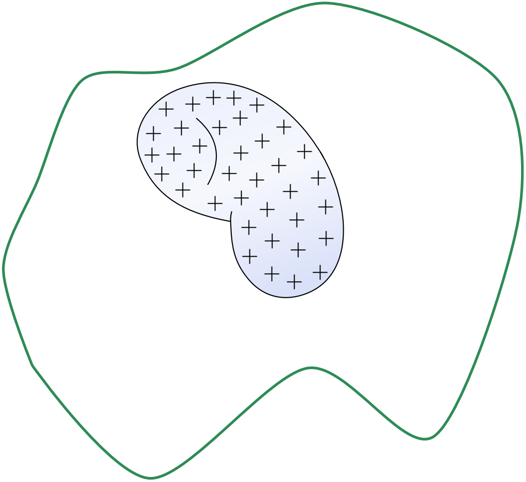
point charge set
arbitrary surface

superposition (continuous)

charge density
arbitrary surface

$$\int_{S'} \vec{E}_A \cdot d\vec{A} \propto \sum_i Q_i$$

$$\int_{S'} \vec{E}_A \cdot d\vec{A} \propto \int_{V_{S'}} \rho(\vec{r}_Q) d^3 r_Q$$



$\sum$ummary

Gauß' law:

The total flux out of a closed surface is equal to the total charge inside, divided by ϵ_0:

$$\int_{\text{closed } S} E_{normal}\, da = \frac{\sum \text{of charges inside } S}{\epsilon_0}$$

Reading list:

- The Feynman Lectures on Physics, Volume II, chs. 2,3;
- JD Jackson, Classical Electrodynamics, 2nd Edition, ch. 1.3;
- Dave Nero, Gauß' Law With Arbitrary Surface, numerical simulation;
- On the magic of the $1/r^2$ interaction;

Ponderables:

- Assume a point source creates a flux of "stuff"

  $$\vec{E} \propto \frac{\vec{e}_r}{r^2} \cdot \cos\phi \cdot \sin\Phi \quad .$$

  Does this field obey Gauß' law?
- How do you interpret "stuff" ejected from a source with flux

  $$\vec{E}' \propto |\vec{r}|^{-\alpha} \quad \text{with} \quad \alpha \neq 2 \quad ?$$